Accepted paper with abstract, SoCG'09

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Minimum Manhattan Network is NP-Complete

Given a set *T* of *n* points in R^2 , a network *G* is said to be a Manhattan network on *T*, if for all *p*, $q \in T$ there exists a Manhattan path, consisting of horizontal and vertical line segments, between *p* and *q* and all its line segments are in *G*. For a given network *G*, let the length of *G*, denoted by L(G), be the total length of all the segments in *G*. For a given point set *T*, the Minimum Manhattan Network Problem is to find a Manhattan network *G* on *T* with minimum L(G). Over the past ten years, whether this problem is NP-complete has been open, and there has been a vast amount of research devoted to the designing of approximation algorithms for this problem.

In this paper, we shall prove that this problem is strongly NP-complete, which implies that there does not exist FPTAS algorithms for this problem unless P=NP. The reduction is from the well-known 3-SAT problem, relying on six different gadgets in the reduction. The validity of the reduction has been confirmed with a computer program.